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# Thermal and Stress Analysis of Numi Decay Pipe Thin Head

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## Introduction and Summary

The Numi decay pipe will use a 72 in diameter end closure consisting of a 1 m diameter, 1/16 in thick aluminum central portion, transitioning to a 3/8 in thick steel head at the larger diameters. This transition is achieved with an explosion-welded aluminum-to-steel flange.

The loads on the head consist of vacuum, and the 10 microsecond energy deposition of the beam. This deposition is repeated every 1.87 seconds.

This analysis shows that the maximum temperature at the center of the head under beam impingement is 70.7 C. The cyclic stress due to thermal expansion/contraction is 12.9 MPa, well within the maximum allowable stress for a fatigue life of 5e8 cycles.

## Geometry

The head geometry was taken from drwg # 8875.114-MD-406865, and associated details.

## Allowable Stresses

### Static Stress Analysis

The rules of the ASME Boiler and Pressure Vessel Code, Section VIII, Div. 2, Appendix 4, "Design based on Stress Analysis", were used in this work. The maximum allowable stress intensities for the materials were taken from Section VIII, Div. 1, rather than Div. II, because Div. I values are slightly more conservative.

For 6061-T6 Al, the maximum allowable stress intensity $S_m$ is 10.5 ksi (72.4 MPa).

For A36 steel, the maximum allowable stress intensity $S_m$ is 14.5 ksi (100 MPa)

Based on these values of $S_m$, the admissible primary and secondary stress limits are established, as shown in Table I.

Near welds, these values will be adjusted by a factor of 0.5 to allow for weld efficiency.

**Table I. Stress Limits – MPa (ksi)**

| Material | Primary Membrane $P_m$ | Primary Bending $P_b$ | Primary Membrane + Bending $P_m + P_b$ | Primary Local Membrane $P_l$ | Primary Local Membrane + Secondary Bending $P_l + Q$ |
|---|---|---|---|---|---|
| **6061 – T6 Al** | **72.4 (10.5)** | **108.6 (15.7)** | **108.6 (15.7)** | **108.6 (15.7)** | **217 (31.5)** |
| **A36 Steel** | **100 (14.5)** | **150 (21.7)** | **150 (21.7)** | **150 (21.7)** | **300 (43.5)** |

Fatigue Analysis

The fatigue analysis was based on the Goodman relation,

$$(\sigma_a/\sigma_{am}) + (\sigma_m/\sigma_u) \leq 1$$

where $\sigma_a$ = stress amplitude
$\sigma_{am}$ = fatigue strength for given cycles at zero mean stress
$\sigma_m$ = mean stress
$\sigma_u$ = ultimate strength = 289 MPa (42 ksi) for 6061-T6

From "Properties of Aluminum Alloys" (J. Kaufman Ed., The Aluminum Association, 1999), the stress amplitude $\sigma_a$ at which 6061-T6 smooth rotating beam specimens survive 5e8 cycles (the maximum tabulated) is 93 MPa (13.5 ksi).

## Energy Deposition

The energy deposition from the beam in the center region of the head is given in Table II.

**Table II. Energy Deposition in 6061-T6 Al Head for One Beam Cycle**

| Radii of Annuli | | Energy (J) |
|---|---|---|
| **Inner Radius (m)** | **Outer Radius (m)** | |
| **0** | **0.001** | **0.360** |
| **0.001** | **0.002** | **0.974** |
| **0.002** | **0.004** | **2.530** |
| **0.004** | **0.006** | **1.830** |
| **0.006** | **0.008** | **0.720** |
| **0.008** | **0.010** | **0.176** |
| **0.010** | **0.012** | **0.030** |
| **0.012** | **0.014** | **0.010** |
| **0.014** | **0.016** | **0.005** |

The total energy deposited in a single pulse is 6.635 J. The time interval over which this energy is delivered is 10 μseconds, giving a heat deposition rate of 663.5 kW.

**The Finite Element Model**

An axisymmetric finite element model of the head was created with ~800 four-node elements. A thermal version of the model was used to calculate the temperature profile in the head; a structural version of the model then read the temperature profile to calculate stresses.

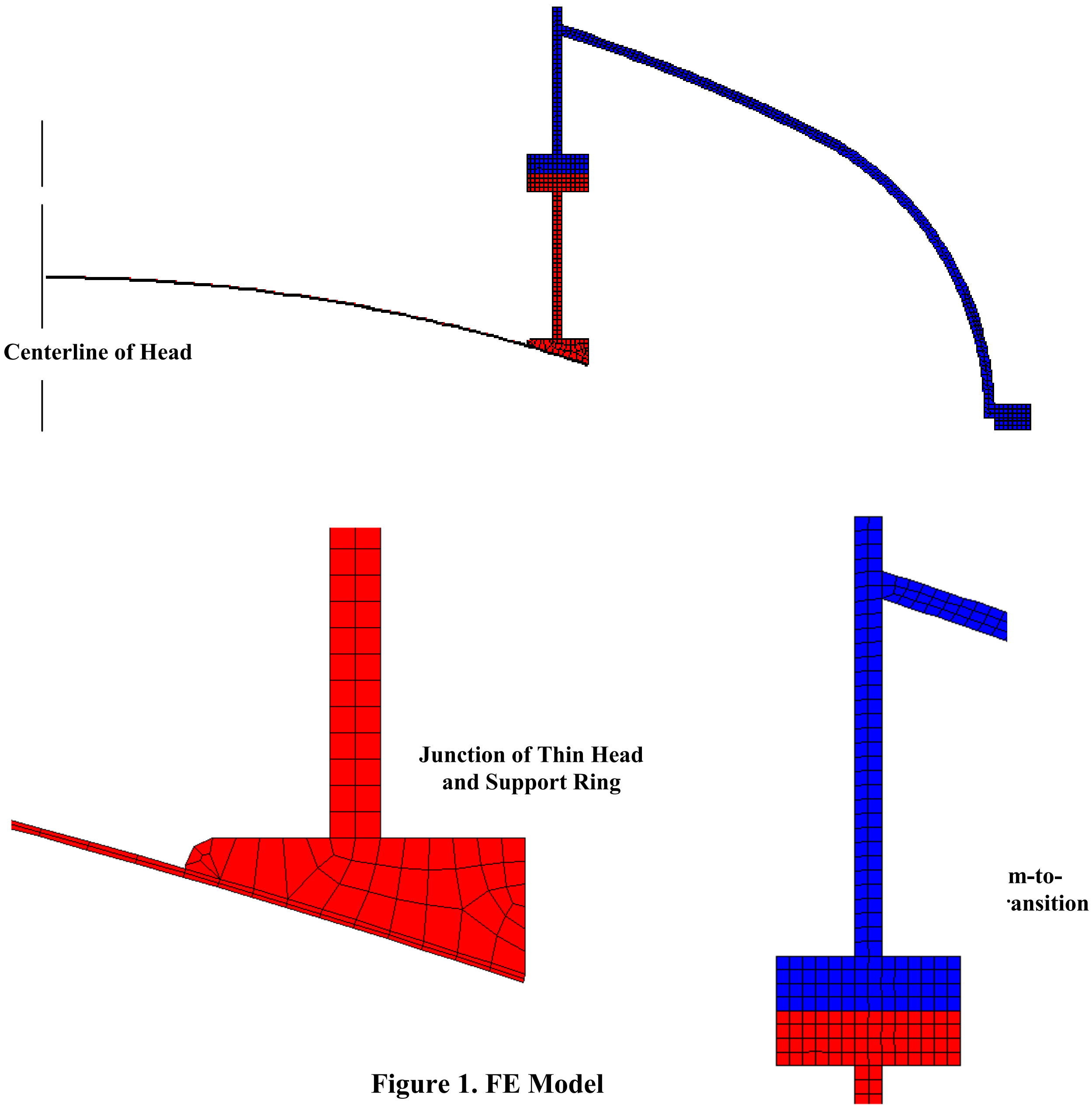



**Figure 1. FE Model**

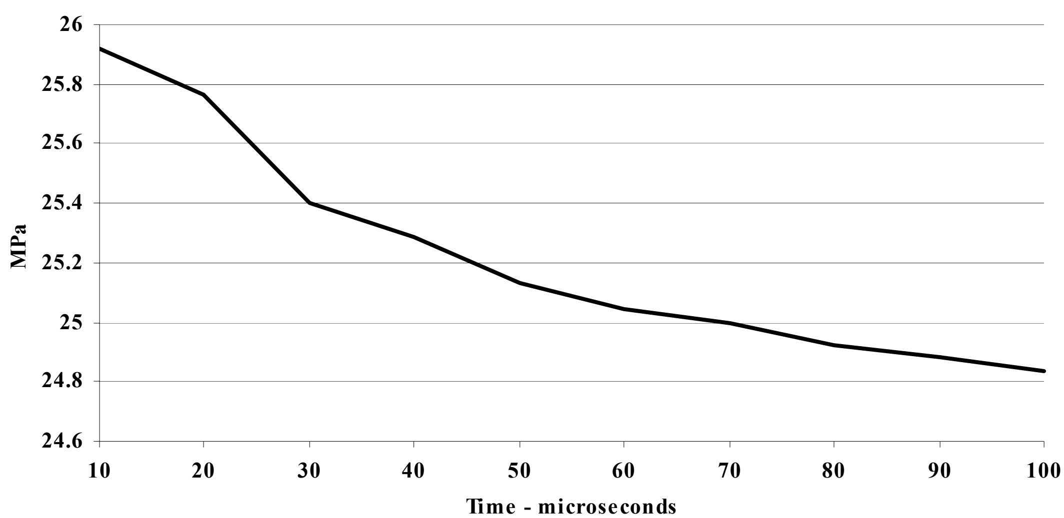
Prior to the final analysis, several runs were made to look at the possibility that significant dynamic effects might result from the sudden heating of the window. The model was refined to a 1 mm element size, and a time step of 2e-7 sec was used to track the stresses as they propagate from the center of the thin head toward the outer radius 0.5 m away.

Fig. 2 shows the stress at the center of the head for the 90 microseconds following one pulse. The stress is a maximum of about 26 MPa at the end of the 10 μsec pulse, and decreases over the remainder of the time interval.

Fig. 3 shows the propagation of the stress wave outward from the center of the head. The wave travels at approximately 5000 m/sec, consistent with sonic velocity in aluminum. The wave amplitude is a maximum of about 0.5 MPa (72 psi) near the center of the head, and attenuates as the wave moves outward radially. The amplitude of 0.5 MPa is negligible on the scale of the 26 MPa maximum stress.

From this work it is concluded that dynamic effects are not significant, and they are not addressed further in this analysis.

**Figure 2. Stress Intensity at Center of Thin Head after 1 Pulse**



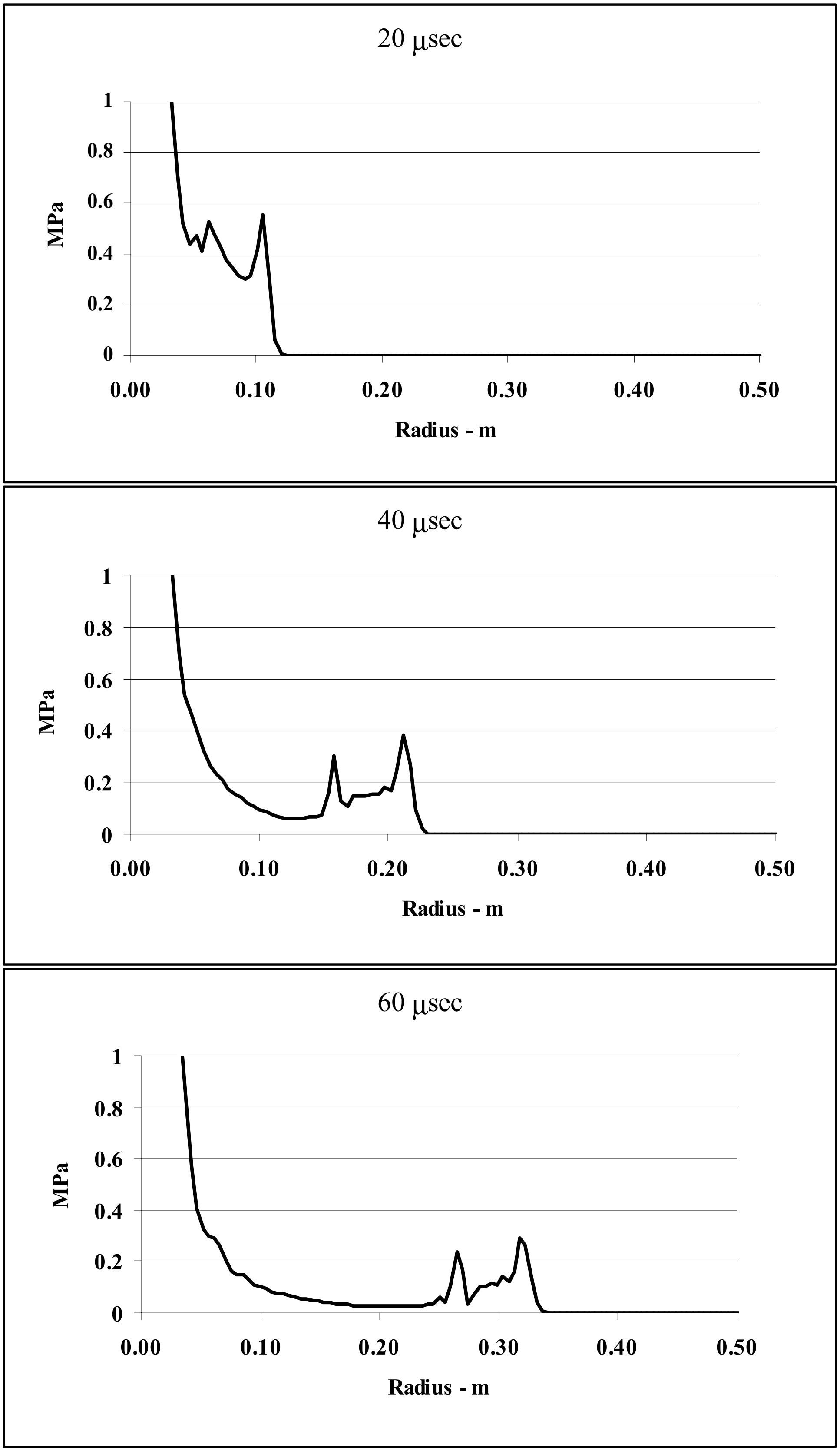



**Figure 3. Stress Wave Propagation in Thin Head**

## Steady-State Thermal Response

The thin window was assumed to have an initial temperature of 30 C. A film coefficient of 5 W/m$^2$ was applied to the outer surface to represent a small amount of natural convective cooling.

When the window is struck by the first 10 μsec pulse, its temperature rises from 30 C to 63 C. During the remainder of the 1.87 second interval, no beam impinges the window, and the center temperature drops back to about 32 C. As pulsing continues, the quiescent temperature continues to rise, as does the maximum temperature at the end of the beam pulse.

Fig. 4 shows the minimum and maximum center temperature rise during 5000 seconds of pulsing. Both temperatures reach their steady-state values after about 2000 seconds. The steady-state minimum and maximum thin head temperatures are 37.3 C and 70.7 C, respectively.

**Figure 4. Temperature Rise at Center of Thin Head**

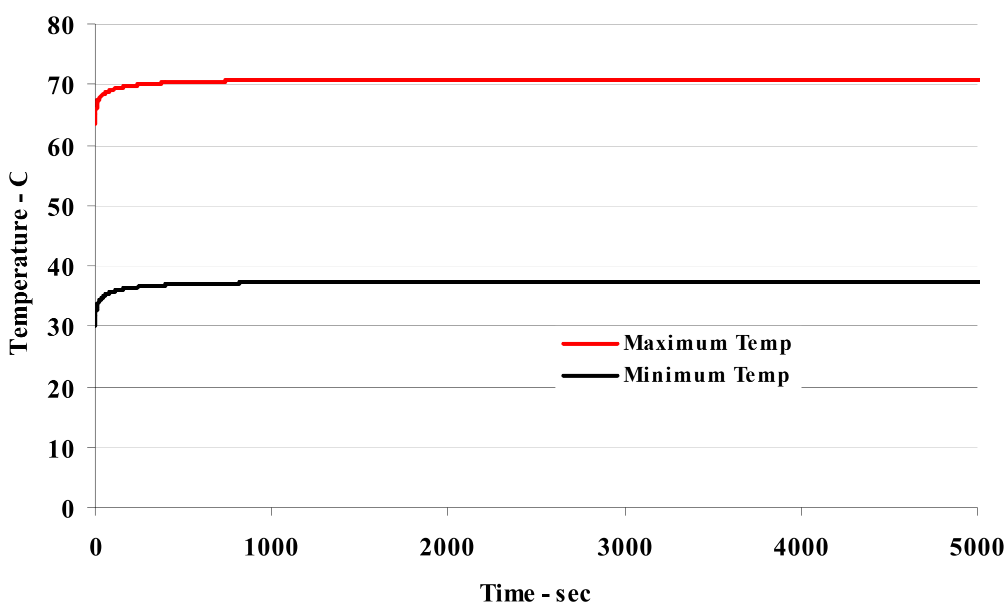
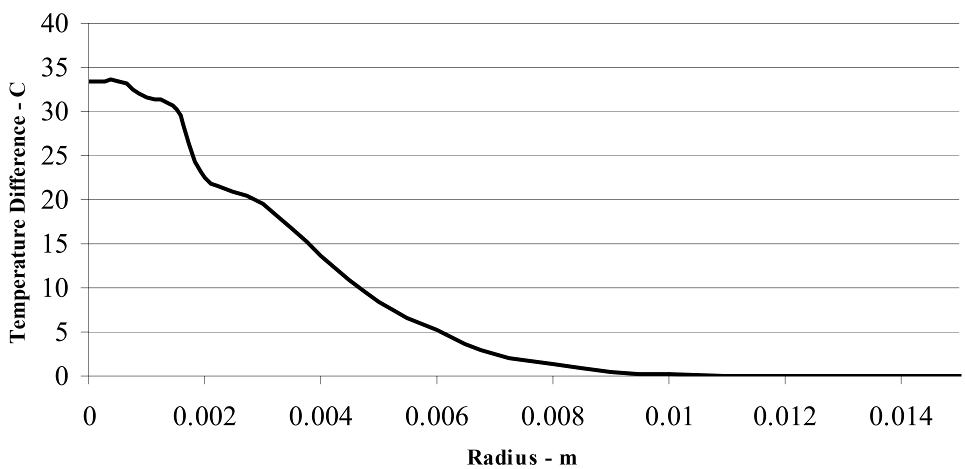


Fig. 5 shows the steady-state temperature difference in the thin head as a function of radius. The plot shows that for radii greater than 1 cm, no significant temperature cycling occurs. Most of the thin head remains at 30 C, as do the remaining steel and aluminum head components.

**Figure 5. Temperature Difference in Thin Head Between Pulses at Steady-State**




## Static Stress Analysis

The steady-state temperatures from the thermal analysis were input to the mechanical FE model, and two sets of stresses calculated. The first set occurs at the end of a 10 μsecond pulse, at which the largest thermal gradients exist. The second set occurs at the end of the 1.87 second quiescent interval, for which the thermal gradients are small.

The model shows that the maximum stresses in the thin head are the result of the 15 psi pressure load; the heating of the head by the beam acts to reduce the stress in the heated area. This can be understood by considering that the head is in a state of tension due to the pressure, but the heated area expands, reducing the tension and thus the stress.

The pressue stress from the FEA can be compared to that calculated from thin shell theory.  A thin spherical shell under internal pressure has a membrane stress of

$$S_m = Pr/2t$$

where $P$ = pressure = 15 psi
$r$ = mean radius of head = 70 in
$t$ = thickness of head = 1/16 in

The membrane stress is then 15(70)/(0.125) = 8400 psi (57.9 MPa). The membrane stress from ANSYS for the case of pressure loading only is 8250 psi (58.3 MPa), which agrees well with the theoretical value.

For static stress evaluation, seven sections through the critical regions were defined, as shown in Fig. 6. Stresses across these sections were linearized, and decomposed into membrane and bending components. These components and their sum can then be compared to the values established previously under "Allowable Stresses."

The center of the head is the only region in which there is a significant variation of stress during a pulse; the remainder of the structure does not react to the heat deposition. Maximum stresses are calculated at the end of a quiescent period that occurs after 5000 seconds of pulsing.

Tables IIa and IIb summarize the stress results. (Note that all sections except A-A are considered regions of secondary stress, $P_l + Q$.)

**Table IIa. Static Stress Results for Thin Head – MPa (ksi)**

| Section (see Fig. 6) | $P_m$ ANSYS | $P_m$ allowable | $P_b$ ANSYS | $P_b$ allowable | $P_m + P_b$ ANSYS | $P_m + P_b$ allowable |
|---|---|---|---|---|---|---|
| **A-A** | **58.3 (8.46)** | **72.4 (10.5)** | **2.5 (0.36)** | **108.6 (15.7)** | **60.8 (8.82)** | **108.6 (15.7)** |

**Table IIb. Static Stress Results for Other Components – MPa (ksi)**

| Section (see Fig. 6) | $P_l$ ANSYS | $P_l$ allowable | $P_l + Q$ ANSYS | $P_l + Q$ allowable |
|---|---|---|---|---|
| **B-B** | **81.8 (11.8)** | **108.6 (15.7)** | **106.9 (15.5)** | **217 (31.5)** |
| **C-C*** | **29.3 (4.25)** | **54.3 (7.8)** | **51 (7.40)** | **108.6 (15.7)** |
| **D-D*** | **2 (0.29)** | **54.3 (7.8)** | **2.5 (0.36)** | **108.6 (15.7)** |
| **E-E*** | **4.4 (0.64)** | **75 (10.9)** | **5.9 (0.85)** | **150 (21.7)** |
| **F-F*** | **48.1 (6.98)** | **75 (10.9)** | **53 (7.67)** | **150 (21.7)** |
| **G-G*** | **5.3 (0.77)** | **75 (10.9)** | **10.4 (1.51)** | **150 (21.7)** |

**Note: Asterisk indicates that section is within 1 inch of a weld.**

The tables show that all stresses are below the allowables. In most cases the difference is substantial – the thin central head region, for example, is stressed to approximately 55% of its maximum. This is a favorable condition with respect to the fatigue analysis, which follows.

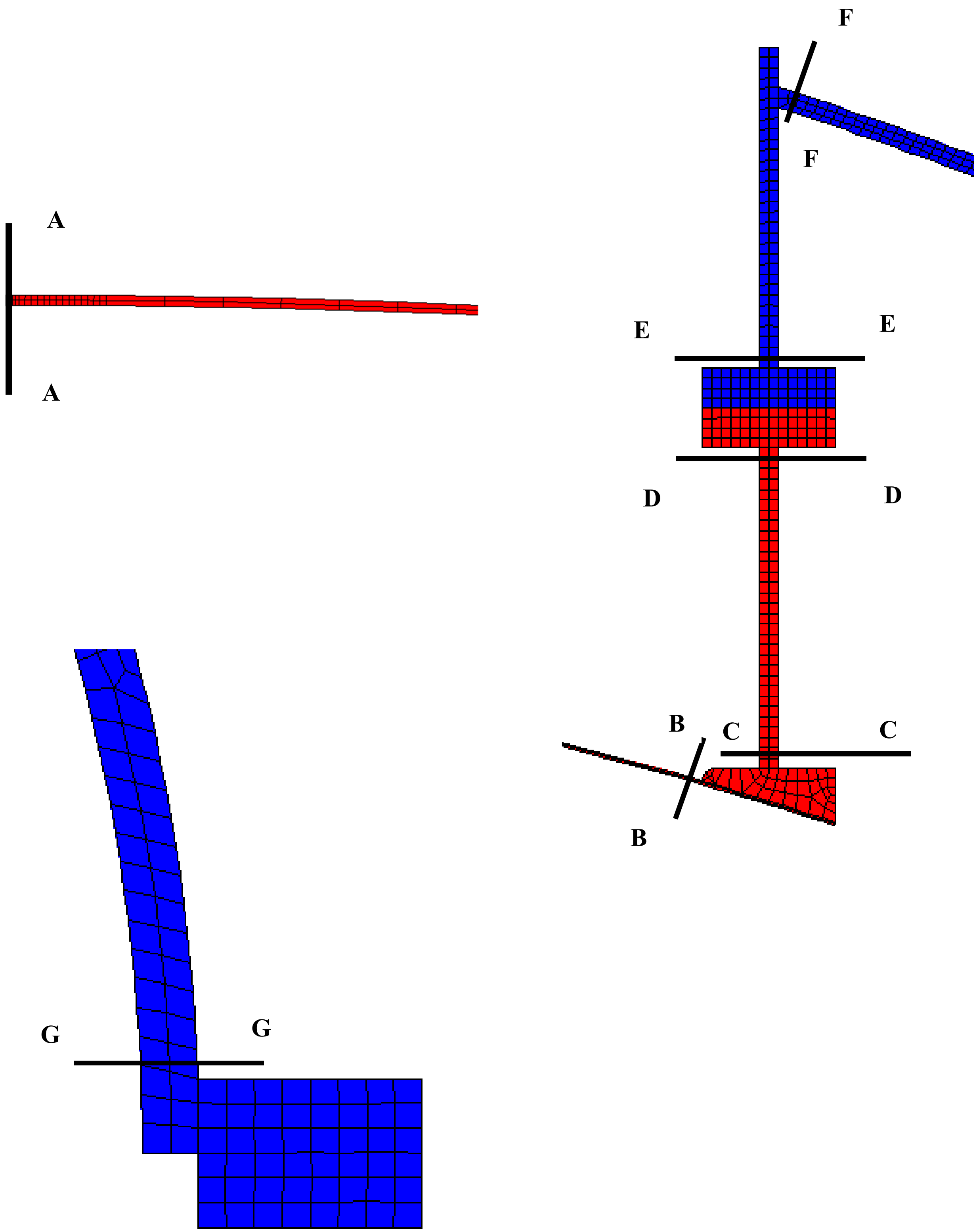



**Figure 6. Sections for Stress Evaluation**

## Fatigue Analysis

The central portion of the thin head is subject to fluctuating stresses. Fig. 7 shows the fluctuation after the head has reached the steady-state thermal condition. The maximum stress is 60.8 MPa (8.46 ksi). The minimum stress is 35 MPa (5.08 ksi). These two stresses will be used with the Goodman relation to determine if the fatigue life will reach or exceed 5e8 cycles.

The mean stress is (60.8+35)/2 = 47.9 MPa (6.95 ksi). The stress amplitude is (60.8-35)/2 = 12.9 MPa (1.87 ksi). From the Goodman relation,

$$(\sigma_a/\sigma_{am}) + (\sigma_m/\sigma_u) \leq 1$$
$$(12.9/93) + (47.9/289) = 0.30 \leq 1$$

Therefore, the thin head satisfies the Goodman criterion for a fatigue life of 5e8 cycles. This is approximately 30 years of continuous beam cycling.

**Figure 7. Fluctuation of Stress in Central Portion of Thin Head**